Predict How Many

Based on Activity 15.14, p. 271

Grade Level: Fifth or sixth grade.

Mathematics Goals

- To explore growing patterns using three representations: pictures or drawings, table of values, and a rule.
- To practice searching for relationships between the step number and the step in a growing pattern as a foundation for the concept of function.

Thinking About the Students

Students have had some experience with growing patterns. They have extended growing patterns with appropriate materials and explained why their extensions followed the patterns. Students have created tables to record the numeric component of patterns (the number of objects at each step). They have found and described recursive relationships (i.e., how the pattern changes from one step to the next). They have not begun to use variables in their explanations.

Materials and Preparation

- Make a copy of both worksheets for each student.
LESSON

BEFORE

Brainstorm:

- Distribute the “Windows” pattern worksheet and display it on the overhead. Explain that the table shows how many bars or sticks are needed to make all of the windows for that step. Have students draw the next two steps and fill in the next two entries of the table.
- Ask: “If we wanted to find how many sticks it would take to have 20 windows—the 20th step—what patterns can we use to help us so that we would not have to draw all of the steps?” Suggest that they look for ways to count the sticks in groupings and try to connect the groupings to the step numbers. Have students work for a while in pairs and then solicit ideas from the class. Here are some possible ideas that students may suggest:
  
a) The tops and bottoms have the same number of sticks as the step number.
  There is one more vertical stick than the step number. \([\text{Step} + \text{Step} + \text{Step} + 1]\)
  
b) There is a square of four sticks then each new step adds three more. That is,
  four plus three times one less than the step number. \([4 + 3 \times (\text{Step} - 1)]\)
  
c) One stick (at either end) then there are as many sets of three as the step number. \([1 + 3 \times \text{Step}]\)
  
d) Looking only at the table and not the drawing: Start with 4 then add 3 one less
time than the step number. This gives the same result as b. Help students make the connection to the drawing.

- For each suggestion that you get, write the idea in a manner similar to the expressions shown here. Notice in these expressions that “Step” is actually a variable and could be
replaced by n or S or any other letter. It is not necessary that students come up with all of these ideas. Erase the ideas that have been suggested.

- Have students use an idea that they like and explain their rule on the “Windows” worksheet. Then have them use the rule to finish the table.
- Pass out the second worksheet, “Predict How Many.”

**The Task**

Determine the number of items in the twentieth step of the pattern on the “Predict How Many” worksheet without filling in the first 19 entries.

**Establish Expectations:**

- Students should extend the pattern for two more steps, making table entries accordingly.
- Students should describe in words the pattern they see in the picture. They should use the picture and/or the table to determine the number of dots in the 20th step.

**DURING**

- Be sure students understand what they did with the “Windows” worksheet before they continue with “Predict How Many.”
- If students are having difficulty finding a relationship, suggest that they look for ways to count the dots without having to count each one. If they use the same method of counting for each step, they should begin to see how their counting method relates to the step numbers. Have them write a numeric expression for each step that matches their counting procedure. For example, step two is 2 x 3, step four is 3 x 4, and so on.
- Once students think they have identified a relationship, make sure they test their conjecture with other parts of the table and picture.
AFTER

- Ask what entry students found for step 20. List all results on the board without comment. The correct result is 420 but do not evaluate any responses.
- Ask students to come to the board to explain their strategies for identifying and extending the pattern. Encourage the class to comment or ask questions about methods of counting the dots or thinking about the rule for step 20.
- For students who use only the table to find a pattern, have the class see how their idea can be related to the drawings of the dots.

ASSESSMENT NOTES

- Are students able to see the connections between the pictorial representation of the pattern and the table of values?
- Look for students who are simply generating all the entries in the table to determine the 20th entry. These students need to be encouraged to look for patterns in the manner that they count the dots.
- If students are ready, you may want to begin to use a letter or variable in the rules that students describe. You may also want to begin to introduce variable notation into their written descriptions (e.g., using “n” for the step number).
### Windows

Name ____________________________________________

![Drawing of windows](image)

<table>
<thead>
<tr>
<th>Step</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of stiks</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Describe the pattern you see in the table:

_________________________________________________________________
_________________________________________________________________
_________________________________________________________________

Describe the pattern you see in the drawing:

_________________________________________________________________
_________________________________________________________________
_________________________________________________________________

Describe how you can find the number of sticks in the 20th step:

_________________________________________________________________
_________________________________________________________________
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Describe the pattern you see in the drawing:

Describe the pattern you see in the table:

Describe how you can find the number of sticks in the 20th step:

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